Climate based daylight simulations with EvalDRC – analysis of Daylight Redirecting Components

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Abstract

The new calculation tool EvalDRC enhances current strategies for the simulation of Daylight Redirecting Components (DRCs). It is built upon RADIANCE as a core simulation engine and uses the daylight coefficient method, climate based sky models for annual simulations, and daylight metrics for data reduction. Contribution Photon Mapping is introduced as a new method for physically correct DRC simulation. Further new features are: separate True Sun Coefficients for a less approximated, more accurate treatment of the sun contribution and monthly breakdowns of the Spatial Daylight Autonomy, and Annual Sunlight Exposure metrics for more detailed information about DRC performance over the course of the year. To show the versatility of the tool, two application studies were carried out in which either measured Bidirectional Scattering Distribution Functions (BSDF) or detailed geometry models were used for the DRC simulation.

Keywords

climate based daylight simulation, daylight metrics, photon mapping, radiance

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1 INTRODUCTION

Daylight Redirecting Components (DRCs) can play an important role in reducing the amount of energy used for artificial lighting by increasing the daylight levels in interior spaces, but a quantitative analysis of their performance needs appropriate strategies. Annual simulations can be run efficiently with the daylight coefficient method (Bourgeois, Reinhart, & Ward, 2008). Light redirection by DRCs can be simulated with measured data of the Bidirectional Scattering Distribution Function (BSDF) (Ward, Mistrick, Lee, McNeil, & Jonsson, 2011). The RADIANCE 3 Phase Method (McNeil & Lee, 2013) and the DAYSIM (Reinhart, 2000) program suite build upon these methods and, to a certain extent, constitute the current standard for annual daylight simulations. Finally, so-called *daylight metrics* (DM) have emerged, which reduce large annual data sets into few meaningful figures for describing the daylight performance. An example is the Spatial Daylight Autonomy (sDA) and the Annual Sunlight Exposure (ASE) proposed by the Illuminating Engineering Society of North America (IESNA) (Heschong et al., 2012).

However, those key technologies do not perform equally well in all conceivable scenarios. Currently used daylight coefficients adequately represent the light from the sky hemisphere, but are less accurate for the sun contribution. This can be problematic in scenes with DRCs, because these are designed to respond to precise sunlight positions. BSDF data sets are not equally appropriate for all available DRC system types, both in terms of their measurement and their use in simulation tools. Finally, annual DMs allow a quick comparison of different installations, but average out valuable information about the DRC performance in different periods of the year. EvalDRC introduces some new concepts to address these limitations.

2 DAYLIGHT SIMULATION STRATEGIES

2.1 DAYLIGHT COEFFICIENTS

2.1.1 Sky coefficients and approximations for the solar contribution

The daylight coefficient method splits an annual simulation into two steps. A light path simulation from the daylight sources throughout the scene produces relative quantities, describing the contribution from different directions to the overall interior lighting. Afterwards, illuminance results are produced by adding up these coefficients, weighted with radiance values calculated from TMY (Typical Meteorological Year) weather data for the chosen timestamps. Sky coefficients can be generated by dividing the hemisphere into 145 *segments* or *patches*, plus one for the ground reflection (Tregenza, 1987). Various ideas have been developed for fitting the sun contribution into that concept. In the RADIANCE 3 Phase Method, it is added to the three sky patches that are closest to the current sun position. To avoid distributing the solar radiance over unrealistically large solid angles, the patches are subdivided by small powers of 2 (Figure 1.A). The DAYSIM tool uses a set of predefined sun positions (currently 65), and averages the actual solar contribution between them (Figure 1.B). As the sun contribution to interior illumination is generally much higher than that of the sky hemisphere, the most valuable contribution is treated in the least accurate way. Therefore, the RADIANCE 5 phase method (McNeil, 2013) introduces a high-resolution sky patch subdivision (division factor 6) and patch-centered predefined suns in a separate step (Figure 1.C).

2.1.2 True sun coefficients

EvalDRC employs the Tregenza subdivision for the sky, while the solar radiance is not distributed or averaged at all. Separate *True Sun Coefficients* are calculated by using exact 0.5° angular sun source primitives for all timestamps of the evaluation period (Fig. 1.D). This guarantees the highest accuracy, theoretically, for the treatment of the solar contribution, at the price of a reduction in flexibility. The True Sun Coefficients are fixed to the timestamps for which they are generated. However, the timestamps are usually fixed according to the requirements of the daylight metrics algorithms, so this drawback can be accepted.

Practical experience has shown that the full accuracy provided by the True Sun Coefficients is not always necessary. Therefore, mainly in the context of calculating results at hourly intervals, as needed in daylight metrics algorithms (Section 2.4), the so-called Shared Sun Path optimization was developed. It is based on the fact that the angular distance of the sun disc between two hours during a day sequence is much greater than that between sun disc positions for the same hour on adjacent days. So, with tolerable accuracy loss, coefficients calculated for the sun path of one day can be used for a few days before and after it (Fig. 2). To take it further, if one takes the 5 day working week into account, the Shared Sun Path concept can be used very efficiently by calculating coefficients only for one day sequence per week (Wednesday), and using this set for the two previous and following days. This reduces the amount of calculations to one fifth of the original count. The maximal angular error between the true and approximated sun positions in this case lies in the order of 1°, and comparisons showed that monthly daylight metrics (Section 2.4.2) differ by less than 1% from those produced with the fully accurate True Sun Coefficients, even for specular mirror profile DRCs with sharp-peaked redirection characteristics. The Shared Sun Path concept thus can be seen as a very moderate approximation, which still retains much of the accuracy of the True Sun Coefficients compared to the coarser methods mentioned in Section 2.1.1.



FIG. 1 Sun coefficient models: RADIANCE 3 Phase Method (A, with subdivision factor 2), DAYSIM (B), RADIANCE 5 Phase Method (C), EvalDRC (D). Only a section of the sky hemisphere is shown.



FIG. 2 Shared Sun Path concept: coefficients calculated for one day sequence (black sun positions, solid light rays) can be used for a few days before and after the calculation day (grey sun positions, dashed light rays)

2.2 BSDF DATA

BSDF data are abstract representations of the light redirection characteristics from incoming to outgoing directions for a specific point on a surface. They can be measured with goniophotometers (Apian-Bennewitz, 2010; Krehel, Kämpf, & Wittkopf, 2015), or calculated with forward ray tracing programs (Grobe, Noback, Wittkopf, & Kazanasmaz, 2015; McNeil, Jonsson, Applefield, Ward, & Lee, 2013). In the simulation calculation, the measured data set is then evaluated with complex algorithms to determine how, and to what extent, the incoming sky- and sunlight is redirected into different directions while passing through a DRC. The approach is covered extensively in literature. One advantage of the method is flexibility. Light flux transfer from the sky, through daylight openings and facade systems, to the work plane in a room can be calculated using a chain of matrices, including a BSDF representation of a DRC. Therefore, different DRCs can be quickly compared in a simulation run by exchanging the DRC matrix in the chain and repeating the matrix multiplication, without needing to repeat the complex light path simulation. This, of course, limits the BSDF data to fixed resolution sets, which are not always appropriate. DRCs with sharp peaks in their redirection characteristics are better represented with flexible BSDF data schemes like the Tensor Tree BSDF (Ward, Kurt, & Bonneel, 2012). EvalDRC supports the RADIANCE BSDF evaluation method, but unlike the RADIANCE 3 and 5 Phase methods, which are limited to fixed resolution BSDFs, EvalDRC can also be used with flexible resolution BSDFs in the Tensor Tree format. A disadvantage of the BSDF data method is the fact that a data set in general is only valid for the point where it was measured. Therefore, for the practical application, BSDF data are appropriate for simulating 'thin' and spatially isotropic DRCs, like prismatic films or double glazings with included specular reflecting lamellae in the gap between the two panes. Extended systems like light pipes, or those for which the redirection characteristic varies strongly across the surface, cannot be easily described with one single BSDF.



FIG. 3 Contribution Photon Mapping: Photons emitted from the light sources are deposited as primary photons upon their first hit in the scene and referenced by each spawned photon. During gathering at point \vec{x} , this reference is used to sort the photons according to the emitting light source.

2.3 CONTRIBUTION PHOTON MAPPING

As a second DRC simulation method and an alternative to the use of BSDF data, EvalDRC introduces the Contribution Photon Mapping technique. It builds upon the general Photon Mapping method (Wann Jensen, 2001), in which rays (photons) are emitted from the light sources. In contrast to the usual backward ray tracing, this allows an accurate simulation of the light particle transport through complex refracting and reflecting media, such as DRCs. A first Photon Map module for RADIANCE was developed in 2004 (Schregle, 2004). Recently, it was enhanced to include support for the contribution coefficient method (Fig. 3) and was integrated into the main RADIANCE distribution in 2016 (Schregle, 2015; Schregle, Grobe, & Wittkopf, 2016). With this approach, DRC geometry models can be directly inserted into a scene for simulation. It is not necessary to have a real sample for BSDF measurement, or to perform a separate calculation step for producing BSDF data from the geometry and material model. In addition, in contrast to the BSDF data method, which does not consider varying properties across a DRC surface, Contribution Photon Mapping is applicable to a wider range of geometries, including extended systems, such as light pipes. However, it imposes greater demands on the accuracy of the used geometry and material model.

2.4 DAYLIGHT METRICS

2.4.1 Annual daylight metrics

In the previous sections, methods for efficient and reliable calculation of annual daylight availability in interior spaces were presented. Usually, the resulting amount of data is far too high to be used directly for comparison or evaluation studies. Here, the Daylight Metrics (DM) come into play. In general terms, DMs are data reduction techniques with respect to both space and time. Several DMs have been developed so far. Among these, the combination of Spatial Daylight Autonomy (sDA) and Annual Sunlight Exposure (ASE) proposed by the IESNA (Heschong et al., 2012) have received widespread acceptance. sDA and ASE characterize the amount of daylight, as well as possible discomfort problems due to sunlight exposure. The sDA specifies the percentage of an analysis area that receives a sufficient amount of daylight (default 300 lux), for a given fraction of the total occupancy time (default 50%). The ASE gives the percentage of the analysis area receiving direct sunlight (default 1000 lux) for more than a specified number of hours per year (default 250 hours). For the occupancy time, a five-day working week with office hours from 08:00 to 18:00 is assumed. Simulation data must be generated at hourly intervals. The illuminance and time fraction thresholds may be adjusted according to the circumstances. For example, for visually challenging tasks a higher illuminance threshold of 500 lux or more can be used. DM definitions rely heavily on assumptions about user behaviour. Heschong et al. (2012) provide extensive discussions and justifications of the method and the proposed thresholds. It is important to note that the sDA must be calculated with dynamic sun shade. This justifies its meaning as useful daylight availability, where situations with overexposure to sunlight no longer occur. Simulation of dynamic sun shading can be very complex. The most generic way, as described in the sDA nominal requirements, is to assume a default sun shade with a low (5-10%) visible light transmittance, which is activated in cases where more than 2% of the analysis area receives more than 1000 lux of direct sunlight. Alternatively, a user defined sun shade geometry (e.g. a venetian blind model) can be used. In practice, this concept needs two EvalDRC calculation runs, with and without sun shade geometry. Afterwards, their results are dynamically mixed in the sDA calculation algorithm according to the given criterion. Final sDA values of 75% are considered as *preferred*, with values above 55% as *nominally accepted* daylight availability.

2.4.2 Monthly daylight metrics

In the official sDA and ASE definition, the time frame is one year, which means a strong reduction of the daylight simulation results to one scalar value each. This allows a quick comparison of different variants according to their overall daylight performance but lacks important information about the performance over the course of the year. It also does not fit well to the general perception of daylight, which, in many regions of the earth, is known as seasonally variant. By changing the time frame from a year to a month, the algorithms for sDA and ASE can be extended to produce additional monthly counterparts of the annual values, denoted as msDA (Monthly Spatial Daylight Autonomy) and MSE (Monthly Sunlight Exposure) (Bauer, 2015). To avoid confusion, it is worth noting that the annual DMs are not just simple averages of the monthly DMs. Examining the monthly DM graphs produced by EvalDRC can deliver valuable information about the advantages and drawbacks of a specific daylight design in different phases of the year. This will be shown in the following application examples.

3 APPLICATION EXAMPLES

3.1 DRC SIMULATION WITH BSDF DATA

EvalDRC was employed in a retrofitting case study for a studio classroom in the department of architecture at the Lucerne University of Applied Sciences and Arts in Switzerland. The room is rectangular, measuring 8.8m x 9.05m, with a south-facing window wall comprising six glazed segments. The Window to Wall Ratio (WWR) is approximately 75%. (Fig. 4.A). The original, almost fully glazed facade produces a high solar gain, so sun shading is often necessary. The aim of the study was to reduce this high solar gain while keeping daylight levels sufficiently high.



FIG. 4 Room with south facing facade (A) and DRC sample made from light redirecting prismatic film (B)

To show how this can be done without introducing expensive construction work on the facade, the use of a light redirecting prismatic film was proposed (Fig. 4.B). The BSDF data approach is very well suited to these type of DRCs, so first its BSDF was measured at the CC EASE goniophotometer lab. This data set was then used in the daylight simulation with EvalDRC. Three optimization scenarios were analyzed, consisting of different combinations of wall, clear glazing and DRC elements. For the simulation, part of the framing was omitted, and the facade was replaced with an abstract representation (Fig. 5.A).



FIG. 5 Schematic layout of the existing facade and three optimization variants (A). Bar diagram showing sDA (black) and ASE (grey) for the existing facade and the three optimization variants 01-03 (B).

The first optimization scenario (01) was simply a reduction of the WWR, achieved by lowering the window height. In the second scenario (02), instead of the WWR reduction, the DRC film was applied to the upper third of each vertical glazing segment. The third scenario (03) was a combination of both measures. Besides the WWR, the DRC to Window Ratio, DWR, is also given. Figure 5.B shows annual sDA and ASE results for the existing facade and the three optimization scenarios, as calculated by EvalDRC based on local TMY weather data. Work plane height was 0.94m. As visually challenging tasks are performed in the room, the sDA illuminance threshold was set to 500 lux. Simply reducing the WWR with O1 lowers the sunlight exposure (ASE), but results in an unsatisfactory sDA. Inserting the DRC in O2 results in a significant sDA increase, but still produces a rather high sunlight exposure. ASE is in fact the same as in variant 01, because the effective window height is equal in both scenarios. The variant O3 clearly shows the best performance. Here, the effect of the WWR reduction is counteracted by the DRC element, resulting in the lowest sunlight exposure (ASE < 6%) and an sDA value of ~80%, which still fulfills the nominally preferred criterion. The study shows that with a simple measure of applying a prismatic film on selected parts of the glazing, the daylight performance in terms of useful daylight, without overexposure to sunlight, can be improved significantly.



FIG. 6 Monthly daylight metrics msDA (A) and MSE (B) for the optimization variants compared to the existing facade.

Figure 6 shows the comparison of the monthly daylight metrics msDA and MSE. The graphs explain in detail how the effect of the three optimization variants builds up during the course of the year. Simply reducing the WWR (01) results in an almost evenly reduced msDA by about 20% for all months, except the summer months June and July. The DRC introduction in 02 instead has its main effect in spring and autumn, and almost no effect during the winter months. Finally, the good performance of the 03 variant does not mean increased daylight availability throughout the whole year. In fact, it is a net effect resulting from a strong to moderate msDA increase in spring, autumn and summer, and a decrease in winter. In addition, the 03 variant produces high daylight availability, spread almost evenly throughout the period from spring to autumn, but produces a sharper drop for the winter months. This can be mainly attributed to the local weather conditions, which are mostly cloudy in the Lucerne area during winter. Thus sunlight redirection, the main effect of a DRC, cannot contribute much during this time. Further details about the case study can be found in Kazanasmaz, Grobe, Bauer, Krehel, & Wittkopf (2016).

3.2 DRC SIMULATION WITH GEOMETRY MODELS

For cases in which BSDF data for a DRC are not available, the new Contribution Photon Mapping module for RADIANCE (Section 2.3), which can be used conveniently through the EvalDRC tool, offers an alternative way of simulating DRCs with geometry models inserted directly into the scene. This opportunity is an important new enhancement to the existing simulation possibilities, and was previously not available in the context of RADIANCE based daylight simulations. It can be especially helpful in the prototyping phase, when a real sample is not yet available, or for geometrically extended DRCs that cannot be easily represented with BSDF data. This feature of EvalDRC was used in a new daylight course at Izmir Institute of Technology (IYTE) in the spring semester of 2016.



FIG. 7 Classroom with south-facing facade used for the project work (A) and combined interior and exterior light shelf with upward bent shape (B)

The central part of the course was a project task in which students had to develop a daylight design strategy from scratch and analyse its performance with both standard lighting simulation software and the new EvalDRC tool for correct DRC simulation. For the project work, a rectangular classroom at the IYTE campus was chosen, 12.5m x 6.5m, with a south-facing facade at the shorter side and a ceiling height of 3.0m (Fig. 7.A). The students were free to select the window size, as well as the wall, floor, and ceiling materials. DRC elements could be added on both the interior and exterior. Some details, like positioning specular blinds in the air gap between double glazed windows, were deliberately avoided. TMY weather data for İzmir was used in the calculations. Just one example from the many different designs is presented here. The proposed DRC consists of a combination of a flat interior and upward bent exterior light shelf with a specularly reflecting upper surface at a height of approximately 2.5 m above the floor (Fig. 7.B). Additionally to the DRC, a sun shade made from small, dark blue colored venetian blinds is inserted in the region below the light shelf.



FIG. 8 Daylight metrics for the classroom scene with DRC (filled bars /solid lines) and the reference room without DRC (hatched bars /dashed lines)

Figure 8 shows the resulting annual and monthly daylight metrics of the room with DRC (filled bars and solid lines) and for the reference room without DRC (hatched bars and dashed lines). Black represents sDA and msDA, while ASE and MSE are printed in grey. In addition, the line plot contains graphs showing the percentage of timestamps with active sun shade (light grey), that resulted from the DM calculation according to the given criteria for sun shade use, as explained in Section 2.4.1. The DM graphs show that the daylight performance of the reference room is below the satisfactory level (sDA ~ 40%), which can be mainly attributed to the high need for sun shading especially during the spring, autumn and winter months. Regions further away from the window can no longer be sufficiently illuminated with daylight when the sun shade is drawn, which is an important aspect here due to the great room depth. Introducing the DRC results in a significant increase of the daylight availability. Interestingly, the rather simple DRC shape produces an almost uniformly high msDA across the whole year. It is most effective in spring and autumn. Besides the redirection of daylight to the ceiling, the question of sun shading again plays an important role here. The DRC leads to a great reduction in the need for active sun shading, which also contributes to the increased daylight availability.

In addition to the daylight metrics, a visualization sequence was produced with EvalDRC, making use of the potential of the Contribution Photon Map module to produce physically realistic images showing the redirection effect caused by the DRC. For the sequence, a clear sky was used, because the focus here was a qualitative analysis of the overall impression produced by the sunlight redirection. Figure 9 shows some examples from the complete sequence. For the period between winter and summer solstices, one image representing 12:00 noon on the 22nd day of each month is shown. Depending on the sunlight exposure, images from the run with and without sun shade are chosen. The images confirm the data presented in the daylight metrics graphs. The greatest light redirection into the depth of the room is happening in the spring and, by symmetry, the autumn months. Furthermore, the effect of the upward bent shape of the exterior part of the light shelf becomes clear. It also produces light redirection into greater room depths during the summer months when the solar altitude in Izmir is very high at noon. This effect cannot be achieved with flat light shelf geometries alone. The visibly distinct reflection pattern stems from the fact that the upward bent shape is modelled not as smooth curvature, but rather as a group of rectangular slats at increasing height and inclination. Finally, the winter images show that limiting the sun shade geometry to the region below the light shelf is not sufficient for very low solar altitudes. In these cases, sunlight from the opening above the light shelf still reaches the work plane at the far end of the room. A possible solution could be made from vertical baffles mounted at a distance from the window greater than the interior light shelf dimension. However, combining light redirection and effective sun shading is, in general, a difficult task for situations in which sunlight shines into the room at low incidence angles.



FIG. 9 Samples from the visualization sequence. One image at 12:00 noon EET resp. EEST for the 22nd day of each month is shown for the December to June period.

4 CONCLUSION

EvalDRC is a comfortable tool for detailed annual DRC simulations based on RADIANCE. It offers two key simulation techniques, BSDF data and Contribution Photon Mapping, and thus is applicable for a wide range of DRC types, ranging from prismatic films or special glazings to extended systems like light shelves and light pipes. In particular the option to simulate DRCs with 1:1 geometry models inserted directly into the scene is an important addition to the available daylight simulation capabilities in the RADIANCE context. The introduction of the True Sun Coefficients offers an alternative, more accurate way for including the solar contribution in the general daylight coefficient method, compared to the existing, more approximated methods. The multiple output formats (visualizations, illuminance values and daylight metrics) provide valuable information for assessing light redirecting effects and their impact on interior daylight availability. The proposed monthly breakdowns of the established sDA and ASE daylight metrics are very helpful, because they contribute detailed information about DRC performance over the course of the year, which helps in identifying advantages and drawbacks of the analyzed daylight designs. In addition, they are more intuitive than the single annual values, because they better reflect the general perception of daylight as a seasonally varying quantity.

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